Tutorial 1
Advanced Graph Theory

24^{th} July 2014

1. State if the following statements are TRUE or FALSE. If false, a counterexample will suffice. If true, a proof is required:
    - 1.1 If every vertex of a simple graph G has a degree 2, then G is a cycle.
    - 1.2 If every vertex of a connected simple graph G has a degree 2, then G is a cycle.
    - 1.3 The complement of a simple disconnected graph must be connected.

2. Prove by induction, that the cycle C_n; $n = 2k + 1, k \geq 0$, can never be a sub-graph of $K_{r,s}$.
3. Given a connected simple graph G and $v \in V(G)$; prove that v has a neighbor in every component of $G - v$. Can you then conclude that no graph has a cut-vertex of degree 1?
4. Let W be a closed walk of length atleast 1 **that does not contain a cycle**. Prove that some edge of W repeats immediately (once in each direction).
5. Let v be a cut-vertex of a simple graph G. Prove that $\bar{G} - v$ is connected.